CN 550 **Spring 2011**

Phase Plane Assignment

What is it about?

DO NOT USE COMPUTERS FOR THIS ASSIGNMENT, EXCEPT FOR WORD PROCESSING

A main purpose of this analysis is for you to be able to make rapid, accurate sketches of phase plane dynamics without resorting to a computer.

Another goal is to give you a sense of exploring a problem mathematically. Computer simulations can test hypotheses, but rarely generate them. In recent years, students have come to rely increasingly – even exclusively – on this mode of analysis. This assignment is a chance for you to think about the strengths and weaknesses of complementary analytic tools.

Specify each case in terms of the set of *all* parameters *a,b,c* that produce that case. The phase plane illustration of each case chooses a specific example of parameters from this case, but this is not the whole story.

Try to explore the entire parameter space, following the model of the class lecture notes.

It would take you a very long time, without much benefit, to do local analysis (eigenvalues, eigenvectors, etc.) at each critical point. Indicate how to do this, with an example calculated for 1–2 characteristic points. In your phase portraits, use local geometry to obtain accurate estimates of how trajectories approach critical points.

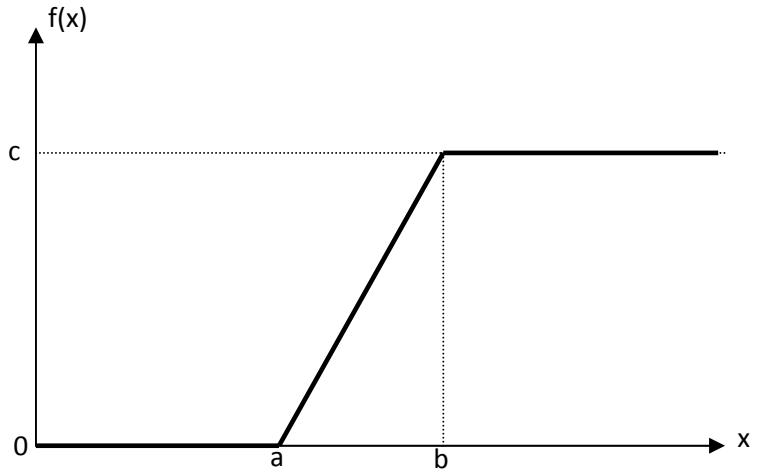
A complete assignment is a coherent *analysis* which is *illustrated* by ~15 phase plane diagrams – *not* an undigested set of plots.

Assignment

Use phase plane analysis to describe the dynamics of the two–dimensional, on–center, off–surround, shunting, competitive network (with inputs ≡ 0) given by:

$$\begin{cases} \dfrac{dx_1}{dt} = -x_1 + f(x_1) - x_1\big(f(x_1) + f(x_2)\big) \\ \dfrac{dx_2}{dt} = -x_2 + f(x_2) - x_2\big(f(x_1) + f(x_2)\big) \end{cases}$$

where $f(x)$ is the piecewise–linear function shown below.



Analyze and show HAND-DRAWN phase portraits of the solutions to this system for characteristic values of the (dimensionless) parameters *a*, *b*, and *c*.

All parameters are ≥ 0, and $a \le b$.

Include details (arrows, etc.) to clarify dynamics, as in the lecture notes.

State the range of parameter values that yield each class of dynamics. For example (hypothetically), "for $0 < ab < c$, all solutions → 0."

Illustrate each class with a characteristic phase portrait, approximately 3" by 3" in size.

Describe your analysis of the phase planes (e.g., nullclines, critical points etc.).