CN510 Assignment 8: Implicit Pair-Based STDP

Due Thursday Nov 14, 2013

The purpose of this assignment is to put plasticity on top of the network from the previous assignment. Use the $C = 0.5$ for the presynaptic traces. If you like, you can run the simulations with $C = 0$ and $C = 1$ for extra credit.

Postsynaptic neuron

Last time you have used

$$x_{i}(n + 1) = x_{i}(n)e^{-A(t_{n+1} - t_{n})} + s_{i}(n + 1)(B - Cx_{i}(n)) \quad (1)$$

to compute presynaptic trace and then added these traces together to get a cumulative trace.

$$x_{cum}(n) = \sum_{i} x_{i}(n) \quad (2)$$

This time we will do a weighted sum

$$x_{cum}(n) = 2 \sum_{i} w_{i}(n)x_{i}(n) \quad (3)$$

Note that the alternative way using a single Rotter-Diesmann trace is also possible,

$$x_{cum}(n + 1) = x_{cum}(n)e^{-A(t_{n+1} - t_{n})} + \sum_{i} s_{i}(n + 1)(B - 2Cw_{i}(n)x_{i}(n)) \quad (4)$$

so once again I ask you to verify that the two produce the same output.

Generate 100 random initial weights $w_{i}(0)$ uniformly distributed between 0 and 1. This weight distribution shall reduce your input by about a factor of 2, hence I multiplied the sum by 2 to get you in the same range of output voltages. Generate a histogram of the initial weights splitting the [0,1] interval into 25 bins.

Use the rest of the network from previous assignment. During the run through on Tuesday we saw voltage level around 30 mV above rest, I presume that most of you will get in this range, but if you can’t without spending too much time, then simply use about average value of voltage that you are getting as an initial value for threshold, we will adjust it anyway.

Convert your postsynaptic cell

$$V(n + 1) = V(n)e^{-D(t_{n+1} - t_{n})} + x_{cum}(n)\frac{T_{s}T_{m}}{T_{m} - T_{s}} \left( e^{-D(t_{n+1} - t_{n})} - e^{-A(t_{n+1} - t_{n})} \right) \quad (5)$$

into integrate-and-fire neuron by adding the spiking threshold $\theta$ so that

- if $V(n + 1) > \theta$ you set $V(n + 1) = -5mV$ and emit a spike (set $q(n + 1) = 1$, use $q(n + 1) = 0$ when there are no postsynaptic spikes).

Note the measuring unit scale again, if you are measuring your voltage in Volts, use reset voltage value of -0.005. Set the threshold to the value discussed above. Run several simulations and adjust the $\theta$ so that with initial weights the cell produces approximately 60 spikes during the simulation time (same time as in the last assignment). List the resulting threshold value in your report.

Plot resulting spike trace against time for the whole simulation. Provide an additional zoom in on the plot for times between 1000 and 2000 ms.
Now add the postsynaptic trace variable. Use the same equation as for presynaptic trace

\[ y(n + 1) = y(n)e^{-A(t_{n+1} - t_n)} + q(n + 1)(B - Cy(n)) \]  

(6)

with \( B = 1, \ C = 0, \) and \( A = 25 \) (we make it slower decaying with time constant 40ms to get closer to K current timings). Plot \( y(n) \) through time for the whole simulation. Provide an additional zoom in on the plot for times between 1000 and 2000 ms. Double check that it matches the spiking from the previous plot.

**Plasticity**

Add pair-based STDP to modify your weights

\[ \Delta w_i(n) = F_+(w_i(n))x_i(n)q(n) - F_-(w_i(n))y(n)s_i(n) \]  

(7)

Pick \( F_+(w_i(n)) \) and \( F_-(w_i(n)) \) as you wish (any combination of additive, multiplicative, or power-based from the lecture) but keep \( w_{max} = 1 \) and learning rate \( \eta = 0.1 \). For extra credit you can do two values of \( \mu \): one below the critical level and one above.

Run the complete simulation with weight updates. Plot resulting weight changes for two of the weights against time for the whole simulation. Provide an additional zoom in on the plot for times between 1000 and 2000 ms.

Take the final values of the weights and generate a histogram of these weights splitting the \([0,1]\) interval into 25 bins. Compare with the initial weights.

Provide the discussion of the correspondence between your expectations and the results you have achieved: weight distribution, firing rate of the postsynaptic cell changes. Note that for some choices the weights can go down for so many inputs that the postsynaptic cell ceases firing before the simulation ends. If this happens to you, provide your explanation and suggest a possible solution.

**Grading Rubric:**

**25 points Setting up the postsynaptic cell**

- 5 points Well-formatted plots with readable labels and parameter meanings listed in the caption
- 10 points Correctness of results
- 10 points Discussion of results including the threshold selection

**20 points Setting up the learning**

- 10 points Discuss your choices of \( F_+(w_i(n)) \) and \( F_-(w_i(n)) \) and how you expect them to affect weight distribution (include discussion of \( \mu \) if you use power STDP)
- 10 points Discuss how the rule can be modified to include axonal or dendritic delays

**55 points Learning Outcome**

- 10 points Well-formatted plots with readable labels and parameter meanings listed in the caption
- 15 points Correctness of results
- 30 points Discussion of results including everything that you have expected but did not achieve in this assignment as well as everything you achieved but did not expect.