Replicate the network of Ermentrout and Kopell, (1998) depicted above. Use a time step of 1ms (use 0.001s in MatLab, run the final simulation for 5s, or 5000 steps). Use the following equation for the leaky integrate-and-fire neuron
\[
\frac{dy}{dt} = -Ay + I_{syn} + I_{ext}
\]
for all four neurons, but set \( I_{ext} = 0 \) for the inhibitory cells. If \( y > \theta \) in the above equation the neuron emits a spike and \( y \) is reset to \( y_{res} = 0 \). Use \( A = 40 \) (equivalent of a membrane constant of 25ms, you can test it if you want by setting initial value \( y = 1 \) and letting it decay, it should be totally gone in 250 steps), \( \theta = 2 \).

**Calibrating Excitatory Neurons** Set \( I_{syn} = 0 \) and manipulate \( I_{ext} \) to achieve a firing rate of about 50Hz. Start with \( I_{ext} = 5 \) and increase or decrease it to adjust the rate. Show a plot of activity from your E cells after calibration and your final choice of \( I_{ext} \) in your report.

To calculate \( I_{syn} \) during the simulation use
\[
\frac{dI_{syn}}{dt} = -BI_{syn} + \sum_{k=1}^{2} w_k \delta(t - s_k - \Delta_k)
\]

Here \( s_k \) is the timing of the spike in the presynaptic cell \( k \), \( \Delta_k \) is the delay between the two cells, and \( w_k \) is the connection weight. Remember that in numerical simulation this means you will be adding \( w_k \) to \( I_{syn} \) (not to its derivative) every time \( (t - s_k - \Delta_k) = 0 \). Set \( B = 200 \) (5ms) for excitatory projections and \( B = 100 \) (10ms) for inhibitory projections.

**Calibrating E-I Interactions** Set cross-circuit connection weights to 0. Set local connections with delay \( \Delta = 1 \)ms (or time step). Start with \( w_k = 1 \) for excitatory projections and \( w_k = -1 \) for inhibitory projections. Adjust excitatory weight so that I cell only spikes once in response to local E cell spike. Show plots from both E and I cells and your final choice of weight in your report (if \( w_k = 1 \) works fine you might want to find a maximal weight that only causes one spike).
Main Course  Enable cross-circuit projections and set your cross-circuit weights to the same values as local weights. For cross-circuit delays use a range of values between 1ms and 50ms. Run the complete network multiple times for different delays. You do not need to run all 50 simulations, the task is to find “critical” values. An example conclusion of your report can say something like “the network synchronizes for the delays above 5ms and below 15ms, goes antiphase above that, and dies out below that”. Add the plots of the examples of different behaviors to your report.

Grading Rubric:

15 points  Calibrating E cells
10 points  Well-formatted plots with readable labels and parameter meanings listed in the caption
5 points  Discussion of interesting trends that you have noticed.
(Hint: how linear is the dependency between $I_{ext}$ and firing rate)

20 points  Calibrating E-I interactions
10 points  Well-formatted plots with readable labels and parameter meanings listed in the caption
10 points  Why did I suggest to pick the largest weight that leads to a single spike rather than any other?

65 points  Main Simulation
30 points  Well-formatted plots with readable labels and parameter meanings listed in the caption
35 points  Discussion of results including critical values of delays for transition between different regimes and description of these regimes.